



## NEW BOOKS

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**Résumé.** Cette rubrique présente de brèves revues de livres qui peuvent avoir un intérêt pour les lecteurs des "Cahiers".

**Abstract.** This "New Books" entry presents brief reviews of books which might be of interest for the readers of the "Cahiers".

**Key words.** Topology, Differential Geometry, Anticipation. Categories.

**MS Classification.** 00-01, 00-02, 97-02.

**1. "Synthetic Differential Topology", by Marta Bunge, Felipe Gago and Ana María San Luis**

This book, published by Cambridge University Press (in the London Mathematical Society Lecture Notes Series 448, 2018), gives a comprehensive coverage of Synthetic Differential Topology (SDT) as an extension of Synthetic Differential Geometry (SDG), after a clear recall (Part I) of the main notions of Topos theory and SDG necessary for the sequel.

In the fifties, the introduction of local and infinitesimal jet bundles by Charles Ehresmann and of 'near points' by André Weil lead to a new foundation of Differential Geometry. In the seventies, F. William Lawvere, followed by Anders Kock, developed SDG by 'synthetically' transposing these notions in a non-boolean topos  $E$  with a commutative unitary ring  $R$  'of line type' which satisfies adequate axioms, so that the infinitesimal jets are representable by algebraic 'tiny' objects (nilpotents).

Part II shows how to extend to SDG the theory of connections and sprays, with results extending the classical Ambrose-Palais-Singer Theorem. The

Calculus of variations is also extended. In these 2 extensions, the differences with the classical case are well stressed, explaining how they influence the results; for instance in the calculus of variations there is no need of the notion of variation, with local being replaced by infinitesimals.

Part III introduces the main concepts of SDT. While the representability of jets by tiny objects of an algebraic nature is at the basis of SDG, in SDT what is essential is the representability of germs (of smooth mappings) by tiny objects of a logical sort introduced and studied by Jacques Penon in the eighties. The intrinsic topology on any object of a topos, introduced by Penon, also plays a main role.

The strength of SDT is revealed in Part IV which gives applications of SDT to the theory of stable germs of smooth mappings including Mather's Theorem, and Morse Theory on the classification of singularities

Parts V discusses what would be a well-adapted model for SDT, in relation with what is a well-adapted model for SDG. The existence of such a model of SDT is important to recover several classical results on smooth manifolds, but also to extend their generality and their conceptual simplicity. An application to unfoldings is given.

Finally, Part VI describes such a model, namely the Dubuc topos  $\mathbf{G}$  which Dubuc introduced in 1979 as a well-adapted model of SDG. It is a Grothendieck topos with a ring  $\mathbf{R}$  of line type, in which the category of smooth manifolds is fully embedded, the embedding sending  $\mathbf{R}$  to  $\mathbf{R}$ , preserving the existing limits and some other constructions.

This seminal book is very important because it opens the way to the relatively new and important domain of SDT, by giving a unified clear exposition of its main results, up to now dispersed in more or less accessible papers. Moreover, its parallel treatment of SDG makes visible the main points of both theories.

This book is well written and only requires a knowledge of the basic notions of Category Theory; for instance its parts on topos represent a plain and concise approach to Topos Theory. Not being a specialist of SDT, I have much appreciated the numerous Remarks which insist on specific

points, for instance allowing to avoid some risk of confusion. I recommend this book both to mathematicians who just look for an overview of SDG and SDT, and to researchers who want to acquire an advanced knowledge of these domains, with full proofs of fundamental theorems. And its different parts could be used as a basis for advanced courses or seminars.

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## 2. *"Transforming the Future: Anticipation in the XXIst century"*, edited by Riel Miller

This book, published by Routledge (2018) presents the results of a UNESCO project, directed by Riel Miller from 2013 to 2018, on Anticipation and "Futures Literacy" (FL), the motivating assumption being that FL should enable a better grasp of complexity. It is reviewed in these "Cahiers" to illustrate how category theory may provide a theoretical support for such a research.

Anticipation is fundamental for futures-thinking and decision-making. However, in these domains there is still a dominant deterministic and reductionist paradigm. To counter it, there is need of a "Discipline of Anticipation" (Chapter 2) which takes account of the complexity and impredicativity of the world to imagine 'novel' futures, instead of 'colonizing' the future by an extrapolation of the present and past.

In Chapter 1, Riel Miller describes Futures Literacy as "an emergent and evolving human capability to identify, design, target and deploy anticipatory assumptions" and to use them to better distinguish constraints, dependencies and even weak signals in the present. The conjecture is that FL development increases awareness for new opportunities emerging in the present and ways to exploit them, thus improving strategic decision-making in the face of uncertainty.

A new general-purpose research tool, called a "Futures Literacy Knowledge Laboratory", has been designed to experimentally test this conjecture. Such an FLL (Chapter 4) consists in gathering a group of people to collectively exchange ideas about the future in relation to a specific problem (for example in Case Study 3, Chapter 5: using the future for local labor markets in Bogotá). The exchange generates collective intelligence knowledge creation processes which lead to share a number of 'archetypal' anticipatory assumptions of different kinds: some concern closed 'optimisation' or 'contingent' futures, others concern more creative futures seizing new opportunities in the present.

The more theoretical Chapter 3 analyses how category theory can give a conceptual support to the preceding results. Already in the eighties, Robert Rosen<sup>1</sup> had promoted the use of category theory to study anticipation. Here the performance of different specific FLL designs is assessed by using the Memory Evolutive Systems (MES) methodology<sup>2</sup> which provides a 'dynamic' categorical framework to study evolutionary multi-level and multi-agent impredicative complex systems.

A FLL is modelled by a (FL-)MES in which its members act as co-regulator agents. The collective knowledge creation processes are modelled by complexification/decomplexification processes (CDP). They lead to the formation of a shared "FL-Archetypal Pattern" as follows:

(i) Each agent  $G$  has its own record  $A_G$  of a concept  $A$ , e.g. an anticipatory assumption, in its landscape and it is reflected to other agents through their exchanges. (ii) A shared multi-faceted 'archetypal' record  $A^*$ , encompassing the diverse meanings of  $A$ , is constructed by a CDP process that adds  $A^*$  as colimit of the  $(A_G)$ . (iii) The construction implies that the  $A^*$ s are connected by 'complex links' (Emergence Theorem) which play the role of "changes in the conditions of change".

The shared FL-archetypal pattern  $AP$  so obtained acts as an engine to construct an evolving macro-landscape in which the agents acting together develop a retrospection and a prospection process to select strategies. These strategies can "use the future in the present" by recognizing weak signals and new opportunities and exploit them in an innovative way.

The Futures Literacy Framework developed in this book proposes a new approach to anticipation and decision-making which takes into account the complexity and impredicativity of the world. The FL-laboratories offer both an analytical tool and an experimental ground to verify its assumptions; and the several Case Studies around the world studied in Chapter 5 illustrate the power of Futures Literacy to cope with uncertainty and to reach creative decision-making.

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<sup>1</sup> Rosen, R. 1985, *Anticipatory Systems, philosophical, mathematical and methodological foundations*. New York: Pergamon.

<sup>2</sup> Ehresmann, A. & Vanbremeersch, J.-P., 2007, *Memory Evolutive Systems: Hierarchy, Emergence, Cognition*, Elsevier.

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1. Sensing and Making-Sense of Futures Literacy: Towards a Futures Literacy Framework (FLF), *Riel Miller*.
2. The Discipline of Anticipation: Foundations for Futures Literacy, *Riel Miller, Roberto Poli and Pierre Rossel*.
3. Towards a Formal Framework for Describing Collective Intelligence Knowledge Creation Processes that 'use-the-future', *Andrée Ehresmann, Ilkka Tuomi, Riel Miller, Mathias Béjean, and J-P Vanbremeersch*.

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4. Futures Literacy Laboratories (FLL) in Practice: An Overview of Key Design and Implementation Issues, *Riel Miller*.
5. The Futures Literacy Laboratory-Novelty (FLL-N) Case Studies, *Stefan Bergheim*. (This part describes 14 case studies all around the world.)

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6. Gaming Futures Literacy: The Thing From The Future, *Stuart Candy*.
7. An Extended Futures Literacy Process: Design Lessons from Measuring Wellbeing, *Stefan Bergheim*.
8. Gender and the Future: Reframing and Empowerment, *Ivana Milojević*.

(There is an Open Access version of this book available at

<https://www.taylorfrancis.com/books/e/9781351047999> )

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