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CORRECTIONS TO: A CONSTRUCTION OF 2-FILTERED BICOLIMITS OF CATEGORIES

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Résumé. Martin Szyld a fait remarquer que le Lemme 1.14 ne tient pas pour une catégorie 2 pré-2-filtrée telle que définie dans notre article. Ici, nous montrons comment résoudre le problème.
Abstract. Martin Szyld pointed out that Lemma 1.14 does not hold for a pre-2-filtered 2-category as defined in our paper. Here we show how to resolve the problem.
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We are grateful to Martin Szyld for pointing out that Lemma 1.14 does not hold for a pre-2-filtered 2-category (Definition 1.1). That lemma was used to prove Lemma 1.20 which is essential for Theorems 2.4 and 2.5. The problem can be resolved as follows.

1. *On page 94, delete Lemma 1.14.*

2. *On page 99, delete Lemma 1.20 and the line preceding it.*

3. *At the bottom of page 99 and on page 100, delete all the material from the beginning of Section 2 and Definition 2.1 down to (but excluding) Lemma 2.2, and replace the deleted material by the following:*

2. 2-Filtered 2-Categories

We refer here to Definition 1.1 of pre 2-filtered.

2.1a Definition. A 2-category $\mathcal{A}$ is defined to be *pseudo 2-filtered* when it satisfies the following three axioms:

FF1. A stronger form of the axiom F1 of pre 2-filtered.

$$\textit{Given}\quad \begin{array}{ccc} & & A \\ & {}^{f_1}\nearrow & \\ E_1 & & \\ & {}_{g_1}\searrow & \\ & & B \end{array}\;,\; \begin{array}{ccc} & & A \\ & {}^{f_2}\nearrow & \\ E_2 & & \\ & {}_{g_2}\searrow & \\ & & B \end{array} \quad \textit{there exists}\quad \begin{array}{ccccc} & & A & & \\ & {}^{f_1}\nearrow & & \searrow^{u} & \\ E_1 & & \gamma_1\Downarrow & & C \\ & {}_{g_1}\searrow & & \nearrow_{v} & \\ & & B & & \end{array}\;,\; \begin{array}{ccccc} & & A & & \\ & {}^{f_2}\nearrow & & \searrow^{u} & \\ E_2 & & \gamma_2\Downarrow & & C \\ & {}_{g_2}\searrow & & \nearrow_{v} & \\ & & B & & \end{array}\;,$$

with γ_1 *and* γ_2 *invertible* 2-*cells.*

F2. Axiom F2 of pre 2-filtered.

F3. Given two 2-cells as in axiom F2, with $B = A$, and $u_1 = v_1$, $u_2 = v_2$, then there is a single 2-cell ε such that the LL-equation in F2 holds with ε in place of both α and β.

Remark Given two 2-cells as in axiom F2, with $B,\ C_1,\ C_2$ all equal to A, and $u_1,\ v_1,\ u_2,\ v_2$ all equal to id_A, then there exists $A \xrightarrow{w} C$ such that $w\,\gamma_1 = w\,\gamma_2$. Note that this Kennison axiom BF2, see Definition 2.6.

Proof. It follows immediately from axioms F2 and F3 that there is an invertible 2-cell $\varepsilon : w_1 \Longrightarrow w_2$, such that $\varepsilon\,\gamma_1 = \varepsilon\,\gamma_2$. Cancelling εg, we deduce that $w\,\gamma_1 = w\,\gamma_2$ with $w = w_1$. □

2.1b Definition. A 2-category $\mathcal{A}$ is defined to be *2-filtered* when it is pseudo 2-filtered, non empty, and satisfies in addition the following axiom.

F0.

$$\textit{Given} \quad \begin{array}{c} A \\ \\ B \end{array} \quad \textit{there exists} \quad \begin{array}{ccc} A & & \\ & \searrow^{u} & \\ & & C \\ & \nearrow_{v} & \\ B & & \end{array} .$$

When $\mathcal{A}$ is a trivial 2-category (the only 2-cells are the identities), axiom F0 is the usual axiom in the definition of filtered category, while our axiom FF1 is equivalent to the conjunction of the two axioms PS1 and PS2 in the definition of pseudofiltered category (cf [1] Exposé I).

As was the case for axiom F1, in the presence of axiom WF3, axiom FF1 can be replaced by the weaker version in which we do not require the 2-cells γ_1 and γ_2 to be invertible.

The following properties of the construction LL follow for pseudo 2-filtered 2-categories and not for pre 2-filtered 2-categories.

4. *The replacements for the deleted lemmas, to appear just before Theorem 2.4, are:*

Lemma Given any pair of equivalent premorphisms $\begin{array}{ccc} & A & \\ {}^{x}\nearrow & & \searrow^{u_1} \\ F & \xi_1\Downarrow & C_1 \\ {}_{y}\searrow & & \nearrow_{v_1} \\ & A & \end{array} \sim \begin{array}{ccc} & A & \\ {}^{x}\nearrow & & \searrow^{u_2} \\ F & \xi_2\Downarrow & C_2 \\ {}_{y}\searrow & & \nearrow_{v_2} \\ & A & \end{array}$, if $u_1 = v_1$ and $u_2 = v_2$, then we can choose a homotopy defined by a single (invertible) 2-cell ε, $w_1 \overset{\varepsilon}{\Longrightarrow} w_2$, $(\varepsilon,\, \varepsilon) : \xi_1 \Rightarrow \xi_2$.

Proof. It follows immediately from axioms F2 and F3. $\square$

Lemma Given two arrows $x \underset{\xi_2}{\overset{\xi_1}{\rightrightarrows}} y$ in FA, if $\lambda_A(\xi_1) = \lambda_A(\xi_2)$ in $\mathcal{L}(F)$, then there exists $A \overset{w}{\longrightarrow} C$ such that $w\,\xi_1 = w\,\xi_2$ in FC.

Proof. Recall the definition of λ_A in Theorem 1.19. By the previous Lemma with $C_1 = C_2 = C$, and u_1, v_1, u_2, v_2 all equal to id_C, it follows that there is a 2-cell $w_1 \stackrel{\varepsilon}{\Longrightarrow} w_2$, such that $\varepsilon\,\xi_1 = \varepsilon\,\xi_2$. Since ε is invertible, it follows that $w_1\,\xi_1 = w_1\,\xi_2$. Compare with the Remark after Definition 2.1a. □

7. *Minor corrections: on the second last line of page 82, delete the apostrophe in "ours"; on line 3 of page 84, replace "grateful to" by "grateful for".*

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