JEAN BÉNABOU (1932–2022):
The man and the mathematician

Francis Borceux

Résumé. Nous esquissons la personnalité de Jean Bénabou, présentons quelques apects importants de son œuvre et dressons une liste de ses publications.
Abstract. We sketch the personality of Jean Bénabou, present some important aspects of his work and provide a list of his publications.
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Jean Bénabou left this world on Friday, February 11, 2022, at the age of 89. Let us bet that the image that we will keep of him will be that of a highly creative mathematician and an extraordinary speaker.

He was indeed an exceptional speaker. Jean Bénabou was not just going to the blackboard to give a talk; he was entering the scene like a resolute actor "assaulting" the stage to perform a tragedy. He was all the time walking from left to right, from right to left, with a severe, almost aggressive look at his audience. You had the impression that he was not just delivering an interesting mathematical message, he was somehow fighting with you to convince you that he was telling the "'holy mathematical truth". He was having his cigarette holder in the left hand, bringing it regularly to his lips while the cigarette was inevitably extinguished. His talks were masterpieces

of clarity. Each of them was prepared up to the last detail; he wanted everyone to follow properly the message that he was delivering. He was so anxious to take all the necessary precautions to be clear, precise and understood, that when the chairman made him observe that his time was over, his last sentence was inevitably something like "I thank you, but unfortunately I did not even have time to tell half of what I wanted to say".

Jean Bénabou was somehow living for mathematics, for category theory. Those who had the chance to visit him in his apartment in Paris or in his country house in La Garde-Freinet, know that he was getting up very early. At five in the morning, he was already doing mathematics, in the quiet atmosphere of an environment where most people were still sleeping. And when yourself were showing up later in the day, you could not escape Jean telling you the ideas on which he had just been working. Mathematics, and particularly categories, were always around, including in the kitchen of his apartment where his white board was hanging, full of proofs and results, carefully calligraphed. He was always ready to share his ideas with you, sorry, to convince you to share his ideas. And the less that one can say, is that a challenge that he often had to face . . . was to have too many ideas.

But in the evening, when the moment did arrive to take some good time around a glass of beer or a bottle of wine, Jean Bénabou could become the life and soul of the party. And when Sammy Eilenberg was himself around the table, you had right to a highly pleasant competition of these two, on who would tell the best Jewish jokes. Jean was very generous in his hospitality, very smooth and friendly, another man than the warrior giving talks. And, at Oberwolfach or at La Garde-Freinet, those who could hold a bat could not escape affronting him in some ping-pong match.

But this quick sketch of the personality of Jean Bénabou would not be honest if not mentioning a more controversial aspect of it. As a matter of fact, Jean Bénabou became with the years always more reluctant to publish papers. Not reluctant to write down his ideas: he wrote many long manuscripts which almost never circulated and that his son Roland intends to give now to some university library. In fact Jean Bénabou did often consider that he still had to wait before publishing, because things could yet be improved. Once he also wrote that he was afraid to be one day short of new ideas, and thus found much more exciting to investigate a new idea when it came, than

spending time on writing down a paper. But from waiting to waiting, many of his beautiful ideas were never published. And when a good idea is in the air, many people bump into it and independently, via similar or different approaches, provide solutions and publish papers. Jean Bénabou could enter a vehement anger when he was unduly interpreting these works as a kind of theft of his ideas, and even more serious, when he was viewing them as a betrayal of his ideas, just because the point of view adopted by other authors denied his own convictions. Jean was passionate; he could be excessive.

For more details and some anecdotes on Bénabou's personality and life, see Jacques Roubaud's text "Esquisse d'un portrait de Jean Bénabou, catégoricien" [32].

Jean Bénabou is born in Rabat, in Morocco, in 1932. He comes to study in France and enters the *École Normale Supérieure* in 1952. His research is next supported by the *Centre National de la Recherche Scientifique*, from 1956 to 1962. Charles Ehresmann is his supervisor in Paris and both are in those days highly interested in the development of pointless topology, in particular the theory of locales, which becomes the topic of the first publication of Jean Bénabou [1]. He gets also interested in another approach of topological and geometrical problems: the Grothendieck toposes.

In 1963, Bénabou introduces (independently from Mac Lane and his coherence theorems [29]), the notion of a *monoidal category*, under the name of "catégorie avec multiplication". This is presented in a *Note* in the *Compte-Rendus de l'Académie des Sciences de Paris* [3], followed by another one in 1964 [4] and a *Note* on relative categories in 1965 [6]. Disregarding the anteriority of the short Bénabou's *Notes*, written in French, the 1965 *La Jolla* paper of Eilenberg–Kelly on *Closed categories* [28] becomes somehow the standard reference for enriched category theory, a fact which seriously irritates Bénabou.

Jean Bénabou presents his *Doctorat d'État* at the "Université de Paris" in 1966, under the title *Structures algébriques dans les catégories*. It is published in 1968 in the "Cahiers de Topologie et Géométrie Différentielle" [9].

The first part of the thesis generalizes theories of Lawvere and Higgins to define multisorted algebraic structures. For that, Bénabou introduces the

notions of types and their associated generic models in relation with Cheval-ley "ćatégories marquées" (p. 48 of the thesis), sheaves and Grothendieck topologies (p. 49) and implicitly Ehresmann structured categories (remark p. viii).

The second part of the thesis introduces a general notion of "binary systems" and their families of associated categories, with the examples of bi-modules and of spans. This frame generalizes the monoidal and relative categories that Bénabou studied earlier.

A very important step in the career of Jean Bénabou is his stay at Chicago, in 1966-67, upon the invitation of Saunders Mac Lane. It is the occasion for him to extend the reflection initiated in his thesis and investigate further pseudo and lax structures. From this, results a well-known paper on *bicategories* and *profunctor/distributors*, published in the Reports of the *Midwest Category Seminar* [7]. This paper is still inspiring and cited today. Various aspects of it clearly influence some of Bénabou's future works on descent and fibered categories, in particular the memoir that he writes with Jacques Roubaud [10].

Back in France, Bénabou organizes a weekly seminar at the *Institut Henri Poincaré*, and later at Jussieu (Université Paris 7). This seminar, together with the Ehresmann seminar (1955–1977), plays a significant role in the de-velopment of category theory in France. Toposes and distributors are among the topics most studied in Bénabou's seminar in the early seventies.

Jean Bénabou is also always ready to accept invitations for coming and teaching his ideas in other universities, something that he does excellently well. And from then on, the course notes written by some auditors of his courses, and published as university preprints, become an important source – sometimes the only one – of access to Bénabou's ideas. This is in particular the case in Louvain-la-Neuve [12] to [15] and [31], and in Darmstadt [23].

Jean Bénabou, as already said, had considered Grothendieck toposes as an efficient algebraic tool to handle topological and geometrical problems. When Lawvere and Tierney introduce elementary toposes, Bénabou imme-diately switches to the study of the internal logical structure of these and to the consideration of the models of (external) algebraic theories in a topos with Natural Number Object [12], [15].

Still largely inspired by the work of Grothendieck, Jean Bénabou starts in the years 1970 to develop what is probably the masterpiece of his mathematical achievement: the theory of fibered categories. He polishes his approach to give it an elegance which reaches beyond what he had done before. In this case, his intense concern of perfection results in his approach being repeatedly improved and almost never published, at the exception of some few aspects [17], [18]. And once more, the publication of the Paré-Schumacher paper [30] on *Indexed categories* becomes somehow the reference to the topic, even if the spirit of the approach is quite different. The less that one can say is that Jean Bénabou is depply affected by this situation and the weak recognition of his own (unpublished) beautiful ideas on fibered categories. For a long time, besides his few early notes, the only access to Bénabou's approach of fibered categories is a beautiful set of notes written (in French) by Jean-Roger Roisin [31], following a course held by Bénabou in 1980 in Louvain-la-Neuve. But Bénabou, in his search of perfection, indefinitely postpones the publication of Roisin's notes, which so never occurs. The author of this bibliographical note has included Bénabou's approach to fibered categories as a chapter in the second volume of his *Handbook of Categorical Algebra*, in 1994.

In the years 2000, another course of Jean Bénabou in Darmstadt results in a collaboration with Thomas Streicher and the publication of preprints [23] and [33], but also a joint paper [24]. Very interesting and original ideas developed by Bénabou in his approach to fibered categories are those of smallness and definability [20]. The almost total ignorance of these beautiful notions in the literature on fibered or indexed categories underlines the fact that Bénabou's ideas on the topic should still remain a source of inspiration. In his permanent search of generalization, Jean Bénabou, around 2012, weakens the notion of fibration to that of foliation [25].

The scope of mathematical interests of Jean Bénabou was very wide, including results on non-standard analysis, on the theory of trees, on the logical foundations of category theory, on the notion of universe in a Grothendieck topos, and so on.

He developed also more philosophically inspired works, like an empiric set theory [19] or an adjunction between *almost* and *very* [26].

A tentative list of Bénabou's Ph.D. students

The topics of the various Ph.D.'s that Bénabou supervised give also evidence of the broadness of his interests. And we suspect that the corresponding tentative list that we provide below is probably not exhaustive.

Brigitte Lesaffre, *Structures algébriques dans les topos elementaires*, thèse de 3ème cycle, Paris VII, 1974.

Jean Celeyrette, *Catégories fibrées et Topoï*, thèse d'état, Paris XIII, 1975.

Yves Diers, *Catégories localisables*, thèse d'état, Paris VI, 1977.

Michel Coste, *Localisation dans les catégories de modèles*, thèse d'état, Paris XIII, 1977.

Marie-Françoise Roy, *Spectre réel d'un anneau et topos étale réel*, thèse d'état, Paris XIII, 1980.

Jacques Penon, *De l'infinitésimal au local*, thèse d'état, Paris VII, 1985.

Dominique Bourn, *La tour de fibrations des n-groupoïdes et la longue suite exacte de cohomologie*, thèse d'état, Paris XIII, 1990.

A tentative list of Bénabou's publications

Our tentative list of papers of Jean Bénabou, together with typescripts of courses that he delivered, is given by the **26 first items in references** below. It is probably not complete.

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Francis Borceux
Institut de Recherche en Mathématique et Physique
Université Catholique de Louvain
2 chemin du Cyclotron
1348 Louvain-la-Neuve (Belgium)
francis.borceux@uclouvain.be