

FOREWORD

In October 2008, an International Conference was organized at the Royal Flemish Academy of Belgium in Brussels to celebrate the 60th birthdays of Francis Borceux and Dominique Bourn. After that, we decided to honour their important contribution to category theory by devoting a volume of the "Cahiers" to Francis Borceux, and a volume of TAC to Dominique Bourn. This is the first issue of the volume for Francis.

Francis obtained his Thesis [2] in 1972, under the direction of René Lavendhomme. Assistant in Leuven from 1970 to 1973, he was appointed Chargé de cours (1973-1980) and Professeur (since 1980) in Louvain-la-Neuve, where he directed 12 theses and, from 1996 to 2001, acted as Dean of the Faculty of Sciences.

We are very grateful to Francis for his role as a highly original researcher in category theory and as one of the most active members of the community in the diffusion of category research for more than 35 years. He regularly organized beautiful category conferences (e.g., the Haute-Bodeux meetings) and summer schools in the Ardennes, where he always took great care to invite not only leading category theorists from all over the world, but also young research students who were given the opportunity to talk about their research. Many generations of mathematicians have greatly benefited from both his scientific guidance and his generous support.

He published, alone or in collaboration, 11 research monographs and textbooks; of particular value is his three-volume comprehensive *Handbook of Categorical Algebra*. He is also an author or co-author of more than eighty research papers, as indicated by his list of publications here below.

The Editors of the volume:

J. Adamek, A.C. Ehresmann, M. Gran, G. Janelidze and R. Kieboom

List of Borceux' publications

Books

- B1. F. Borceux and G. Van den Bossche, Algebra in a localic topos with applications to ring theory, *Lecture Notes in Mathematics* 1038, Springer, 1983, 240 pages
- B2. F. Borceux, *Invitation à la géométrie*, CIACO, Louvain-la-Neuve, 1986, 730 pages
- B3. F. Borceux, *Fasci, logica e topoi*, Quaderni dell'Unione Matematica Italiana 34, Pitagora Editrice, Bologna, 1989, 299 pages
- B4. F. Borceux, LaTeX, *la perfection dans le traitement de texte*, ARTEL, Bruxelles, 1989, 185 pages
- B5. F. Borceux, *Teoria dei fasci su uno spazio topologico*, Università di Genova, 1991, 108 pages
- B6. F. Borceux, *A handbook of categorical algebra*, volume I, Basic category theory, Cambridge University Press, 1994, 345 pages
- B7. F. Borceux, *A handbook of categorical algebra*, volume II, Categories and structures, Cambridge University Press, 1994, 435 pages
- B8. F. Borceux, *A handbook of categorical algebra*, volume III, Categories of sheaves, Cambridge University Press, 1994, 523 pages
- B9. F. Borceux and G. Janelidze, *Galois theories*, Cambridge University Press, 2001, 341 pages
- B10. F. Borceux and D. Bourn, *Mal'cev, Protomodular, Homological and Semi-abelian Categories*, Kluwer Academic Publishers, 2004, 479 pages
- B11. F. Borceux, *Algumas teorias de Galois dos corpos e dos anéis*, Coimbra University Press, 2004, 174 pages

Editions

- E1. F. Borceux (ed.), Categorical algebra and its applications, *Lecture Notes in Mathematics* 1348, Springer, 1988, 375 pages
- E2. F. Borceux (ed.), Actes du colloque en l'honneur du soixantième anniversaire de René Lavendhomme, *Bulletin de la Société Mathématique de Belgique*, série A, XLI-2, 1989, 468 pages

Articles

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6. F. Borceux, Une notion de V-limite, in "Colloque sur l'algèbre des catégories. Amiens-1973", *Cahiers Top. et Géom. Diff.* XIV-2, 1973, 162
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8. F. Borceux, Inductive vector topologies, *Annales de la Société Scientifique de Bruxelles* 88-II, 1974, 201-204
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**COVERING MORPHISMS AND NORMAL EXTENSIONS IN
GALOIS STRUCTURES ASSOCIATED WITH
TORSION THEORIES**

*by Marino GRAN and George JANELIDZE**

Dedicated to Francis Borceux on the occasion of his 60th birthday

ABSTRACT. Nous étudions les revêtements et les extensions normales relatives aux structures galoisiennes munies de ce que nous appelons *foncteurs test*. Ces foncteurs apparaissent naturellement dans les structures galoisiennes associées aux théories de torsion dans les catégories homologiques. Sous des hypothèses additionnelles appropriées, tout morphisme à noyau sans torsion est un revêtement, et tout revêtement est une extension normale, pourvu qu'il soit un morphisme de descente effective. Nos contre-exemples, qui montrent l'importance de ces conditions supplémentaires, sont semi-abéliens, et proviennent de la théorie des groupes, en faisant intervenir des produits semi-directs de groupes cycliques. Nous comparons nos nouveaux résultats avec ceux connus pour les revêtements localement semi-simples et pour les extensions centrales généralisées.

ABSTRACT. We study covering morphisms and normal extensions with respect to Galois structures equipped with what we call test functors. These test functors naturally occur in Galois structures associated with torsion theories in homological categories. Under suitable additional conditions, every morphism with a torsion free kernel is a covering, and every covering is a normal extension whenever it is an effective descent morphism. Our counter-examples showing the relevance of those additional conditions are semi-abelian, and moreover, group-theoretic, involving semidirect products of cyclic groups. We also briefly compare our new results with what is known for the so-called locally semi-simple coverings and for generalized central extensions.

Introduction

The purpose of *categorical Galois theory* is to study covering morphisms in general categories defined with respect to so-called Galois structures,

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