

topology in a topos \mathcal{E} and prove a sufficient condition for the associated classifier of dense subobjects to weakly generate \mathcal{E} . We then concentrate on pre-cohesive geometric morphisms $p : \mathcal{E} \rightarrow \mathcal{S}$ with Boolean \mathcal{S} . We show that if the subobject classifier of \mathcal{E} is connected (Sufficient Cohesion) then \mathcal{E} is weakly generated by the classifier of $\neg\neg$ -dense subobjects.

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1. Weak generation

In [7], Lawvere recalls having read that “the basic program of infinitesimal calculus, continuum mechanics, and differential geometry is that all the world can be reconstructed from the infinitely small” and then proposes a mathematical formulation of the idea that a topos may be generated by a single object T “which in some of several senses is infinitely small. Of course T is not just a single point; but it may *have* only a single point, or more generally the set of components functor may agree with the functor represented by 1 on T and its products and sums”. This proposal is refined in Section VII of [8] and we elaborate on that.

Recall that a geometric morphism $s : \mathcal{E} \rightarrow \mathcal{L}$ between toposes is *connected* if its inverse image $s^* : \mathcal{S} \rightarrow \mathcal{E}$ is full and faithful. In Section VII of [8] the following concept is introduced.

Definition 1.1. Given a connected morphism $s : \mathcal{E} \rightarrow \mathcal{L}$ of toposes, let j in \mathcal{E} be the strongest localness operator for which every s^*Y (for Y in \mathcal{L}) is a j -sheaf. If j is actually the identity map on the truth-value space, then \mathcal{E} is *weakly generated by s* .

In other words, \mathcal{E} is weakly generated by a connected $s : \mathcal{E} \rightarrow \mathcal{S}$ if the smallest subtopos containing the full subcategory $s^* : \mathcal{L} \rightarrow \mathcal{E}$ is the whole of \mathcal{E} . The next example is Proposition VII.6 in [8].

Example 1.2 (The topos of reversible graphs is weakly generated by loops). Let M be the four-element monoid of endofunctions of a two-element set. Collapsing the two constant maps in M determines a quotient morphism of monoids $M \rightarrow N$ to a three-element monoid. This quotient induces a (hyper-)connected geometric morphism $s : \widehat{M} \rightarrow \widehat{N}$ between the associated

